B.Sc. (Hons) 5th Semester Examination,
December–2022

MATH

Paper–BHM-352

Groups and Rings

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt any five questions taking **one question** from each unit. **Question No. 9 is compulsory.***

Unit–I

1. (a) Prove that the set $G = \{a + b\sqrt{2}\, ; a, b \in Q\}$ is an abelian group with respect to addition. 6

(b) Prove that arbitrary intersection of subgroups of a group is a subgroup. 6

2. (a) Prove that G = {1, 2, 3, 4, 5, 6} with binary operation x_7 is cyclic. 6

(b) A subgroup H of a group G is normal if and only if each left coset of H in G is a right coset of H in G. 6

Unit–II

3. (a) Show that an infinite cyclic group G is isomorphic to the additive group of integers. 6

(b) If a group G has a non-trivial automorphism, then prove that group G has atleast three elements. 6

4. (a) Let Z (G) be the centre of a group G. If G/Z is cyclic, then prove that G is abelian group. 6

(b) Write all the elements of symmetric group S_3, as product of disjoint cycles. 6

Unit–III

5. (a) Prove that a ring R in which $x^2 = x \;\forall x \in R$, must be commutative. 6

(b) Define centre of a ring. Prove that centre of a ring R is a subring of R. 6

6. (a) Prove that a division ring is a simple ring. 6

(b) Let S and T be two ideals of a ring R. Then prove that $(S + T) / S \cong T / (S \cap T)$. 6

Unit–IV

7. (a) Prove that the set of all units in a commutative ring R with unity element forms an abelian group with respect to multiplication. 6

(b) An element in a principal ideal domain is prime element iff it is irreducible. 6

8. (a) If F is a field, then prove that F[x] is a principal ideal domain. 6

(b) Let R be a unique factorization domain. An element in R is prime iff it is irreducible. 6

Unit–V

9. (a) If every element of a group is its own inverse, then show that the group is abelian. 2

(b) Prove that intersection of two normal subgroups of a group is also a normal subgroup. 2

(c) Prove that identity mapping is the only inner automorphism for an abelian group. 2

(d) Write the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 4 & 8 & 6 & 9 & 7 & 5 \end{pmatrix}$ as the product of disjoint cycles. 2

(e) Prove that every homomorphic image of a commutative ring is commutative. 2

(f) Show that the $x^3 - 6x + 2$ polynomial is irreducible over Q. 2